

Noncommutative QED+QCD and β -function for QED

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Abstract

QED based on θ -unexpanded noncommutative space-time in contrast with the noncommutative QED based on θ -expanded U(1) gauge theory via the Seiberg-Witten map, is one-loop renormalizable. Meanwhile it suffers from asymptotic freedom that is not in agreement with the experiment. We show that QED part of $U_\star(3) \times U_\star(1)$ gauge group as an appropriate gauge group for the noncommutative QED+QCD, is not only one-loop renormalizable but also has a β function that can be positive, negative and even zero. In fact the β function depends on the mixing parameter δ_{13} as a free parameter and it will be equal to its counterpart in the ordinary QED for $\delta_{13} = 0.367\pi$.

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1 Introduction

The possibility of a noncommutative (NC) space-time coordinates was arisen in the string theory [1, 2]. Thereafter, the theoretical and the phenomenological aspects of the noncommutative field theory (NCFT) has been extensively studied by many physicists [3]. With respect to the phenomenological point of view the main question is the value of the NC-parameter that is about $1TeV$ in recent works [4]. Although these bounds on the NC-parameter are usually obtained by considering the NCFT at the tree level, for those works at the higher orders or from the theoretical point of view the renormalizability of the model is a crucial question. The $U(n)$ gauge field theories in the NC space-time are one loop renormalizable [5] but at the higher order the UV/IR mixing causes such theories to be nonrenormalizable [6, 7]. Meanwhile to avoid the UV/IR mixing, one can construct a θ -expanded NC-quantum field theory via the Seiberg-Witten map [8]. Although the pure gauge sector of such a theory is renormalizable [9], it is shown that in general the noncommutative QED cannot be renormalized by means of the Seiberg-Witten expansion even at the one loop [10]. Therefore it seems that θ -unexpanded NCQED is more safe for the phenomenological purposes. Unfortunately in this case, even in the vanishing limit of the parameter of noncommutativity, one encounters an asymptotic freedom for QED [7, 11] where the increasing coupling with increasing energy is well tested experimentally. Furthermore, for the θ -unexpanded gauge groups in the NC space-time one meets [12]:

- Only the $U(n)$ gauge groups in the NC space-time ($U_{\star}(n)$) are allowed.
- The fundamental and antifundamental representation of $U_{\star}(n)$ can only be occurred. For example, in the $U_{\star}(1)$ case, for arbitrary fixed charge q only the matter fields with charges $\pm q$ and zero are permissible [7, 12].
- When we have a gauge group constructed by the direct product of several simple gauge groups, a matter field can be charged only under two of them.

Therefore, for instance, to construct the standard model of the particle physics (SM) with $SU(3) \times SU(2) \times U(1)$ as the gauge group containing the quarks, one needs additional consideration. It is shown in reference [13] how one can consistently reduce $U_{\star}(3) \times U_{\star}(2) \times U_{\star}(1)$ to the usual SM gauge group through a two-step appropriate Higgs mechanism. The negative β -function for NCQED was obtained for matter fields with ± 1 and zero charges besides pure QED interaction. But it is well known that in the NCFT different gauge fields not only have self interactions (three and four photon vertices in NCQED) but also they can have interactions among each other (for instance gluon-photon interaction). Therefore one can ask if the β -function for NCQED after the full consideration is really negative? In fact to answer to this question we need to consider all particles (leptons as well as quarks) and all gauge bosons. For this purpose to obtain the β -function for QED we explore the quantization of QED+QCD in the NC space-time and ignore the weak interaction for simplicity.

This paper is organized as follows: In Sect. 2 we give a brief review on the $U_\star(3) \times U_\star(1)$ model. In Sect. 3 we explore the quantization of the model and then we calculate the QED β -function in Sect. 4. We summarize our results in Sect. 5.

2 $U_\star(3) \times U_\star(1)$ model

In this section we review briefly the $U_\star(3) \times U_\star(1)$ model which is introduced in [13]. The $U_\star(3) \times U_\star(1)$ theory is described by one gauge field valued in the $U_\star(1)$ algebra, B_μ , and $U_\star(3)$ -valued gauge fields as

$$G_\mu(x) = \sum G_\mu^A T^A, \quad (2.1)$$

in which the generators T^a , $a = 1, \dots, 8$, are the Gell-Mann matrices and $T^\circ = (1/6)^{1/2} \times I_{3 \times 3}$. The finite local transformations of the gauge fields are :

$$\begin{aligned} B_\mu &= V \star B_\mu \star V^{-1} + i/g_1 V \star \partial_\mu V^{-1}, \\ G_\mu &= U \star G_\mu \star U^{-1} + i/g_3 U \star \partial_\mu U^{-1}, \end{aligned} \quad (2.2)$$

where $V \in U_\star(1)$ and $U \in U_\star(3)$ and the \star -product is defined as:

$$f \star g(x, \theta) = f(x, \theta) \exp\left(\frac{i}{2} \overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu\right) g(x, \theta), \quad (2.3)$$

where $\theta^{\mu\nu}$ is an antisymmetry tensor that denotes the noncommutativity of space time through

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}. \quad (2.4)$$

Then the invariant action of the $U_\star(3) \times U_\star(1)$ Yang-Mills theory is

$$S_{NCYM} = -1/4 \int d^4x (B_{\mu\nu} B^{\mu\nu} + \text{Tr}(G_{\mu\nu} G^{\mu\nu})), \quad (2.5)$$

in which

$$\begin{aligned} B_{\mu\nu} &= \partial_{[\mu} B_{\nu]} + ig_1 [B_\mu, B_\nu]_\star, \\ G_{\mu\nu} &= \partial_{[\mu} G_{\nu]} + ig_3 [G_\mu, G_\nu]_\star. \end{aligned} \quad (2.6)$$

The independent matter fields that can be accommodated in this model are electron in the anti-fundamental representation of $U_\star(1)$, up quark in the fundamental representation of $U_\star(1)$ and in the anti-fundamental representation of $U_\star(3)$, and down quark in the anti-fundamental representation of $U_\star(3)$. Meanwhile the electron neutrino can be accommodated in the adjoint representation of $U_\star(1)$. Thus the gauge transformation properties of the

fermions are

$$\begin{aligned}
\Psi_e(x) &\rightarrow \Psi_e(x) \star V^{-1}(x), \\
\Psi_{\nu_e}(x) &\rightarrow V(x) \star \Psi_{\nu_e}(x) \star V^{-1}(x), \\
\Psi_u(x) &\rightarrow V(x) \star \Psi_u(x) \star U^{-1}(x), \\
\Psi_d(x) &\rightarrow \Psi_d(x) \star U^{-1}(x).
\end{aligned} \tag{2.7}$$

The gauge invariant action of this model is

$$\begin{aligned}
S_{NCYM} = \int d^4x [&\bar{\psi}_e \star \gamma^\mu D_\mu^1 \psi_e + \bar{\psi}_{\nu_e} \star \gamma^\mu D_\mu^1 \psi_{\nu_e} + \bar{\psi}_u \star \gamma^\mu D_\mu^{1+3} \psi_u + \bar{\psi}_d \star \gamma^\mu D_\mu^3 \psi_d \\
&- 1/4 (B_{\mu\nu} B^{\mu\nu} + 2Tr(G_{\mu\nu} G^{\mu\nu}))],
\end{aligned} \tag{2.8}$$

in which

$$\begin{aligned}
D_\mu^1 \psi_e &= \partial_\mu \psi_e - \frac{i}{2} g_1 \psi_e \star B_\mu, \\
D_\mu^1 \psi_{\nu_e} &= \partial_\mu \psi_{\nu_e} - \frac{i}{2} g_1 [\psi_{\nu_e}, B_\mu]_\star, \\
D_\mu^3 \psi_d &= \partial_\mu \psi_d - \frac{i}{2\sqrt{6}} g_3 \psi_d \star G_\mu^\circ - \frac{i}{2} g_3 \psi_d \star G_\mu^a T^a, \\
D_\mu^{1+3} \psi_u &= \partial_\mu \psi_u + \frac{i}{2} g_1 B_\mu \star \psi_u - \frac{i}{2\sqrt{6}} g_3 \psi_u \star G_\mu^\circ - \frac{i}{2} g_3 \psi_u \star G_\mu^a T^a.
\end{aligned} \tag{2.9}$$

Moreover $U_\star(n)$ group can be decomposed as follows

$$U_\star(n) = U_n(1) \star NCSU(n), \tag{2.10}$$

where $U_n(1)$ consists the Abelian θ -independent elements of $U_\star(n)$ while $NCSU(n)$ consists the remaining parts. In other words the elements of $U_\star(n)$ can be uniquely written as

$$U(x, \theta) = e^{i\epsilon_0(x)1_n} \star e^{i\epsilon_1(x, \theta)1_n + i\epsilon_a(x, \theta)T^a}, \tag{2.11}$$

in which the first exponential factor is a θ -independent function and by multiplying two different elements of $U_\star(n)$ one can easily show that this factor group is isomorphic to the usual commutative local gauge group $U_n(1)$. Therefore this one-dimensional representation of $U_\star(n)$ can be considered to rewrite the gauge potential $A(x, \theta)$ as

$$A(x, \theta) = A_0(x)1_n + A_1(x, \theta)1_n + iA_a^0(x, \theta)T^a, \tag{2.12}$$

where one can show that under $U_\star(n)$ -transformation, $A_0(x)$ transforms as the usual $U(1)$ gauge field. In other words for the first step symmetry breaking a commutative scalar field (which was called Higgsac) is enough to reduce the extra $U(1)$ -field. In fact in the $U_\star(3) \times U_\star(1)$ gauge theory there exist two $U(1)$ factors which can be reduced to one through an appropriate Higgs mechanism. This extra scalar field is charged under the both

sub-groups, $U_1(1)$ and $U_3(1)$. The gauge transformation of the Higgsac field for the first symmetry breaking is

$$\phi(x) \rightarrow U_3(x)\phi(x)V_1^{-1}(x), \quad (2.13)$$

where $U_3(x) \in U_3(1)$ and $V_1(x) \in U_1(1)$, also here $\phi(x)$ is θ -independent and the multiplication is in the commutative space. Therefore the gauge invariant terms including the Higgsac field are

$$D_\mu^{1+1}\phi(x)D_\mu^{1+1}\phi(x) + m^2\phi^\dagger(x)\phi(x) - \frac{F}{4!}(\phi^\dagger(x)\phi(x))^2, \quad (2.14)$$

where

$$D_\mu^{1+1} = \partial_\mu + i\frac{3g_3}{\sqrt{6}}G_\mu^\circ - i\frac{g_1}{2}B_\mu. \quad (2.15)$$

Here B_μ and G_μ° are the θ -independent parts of their corresponding noncommutative fields. After the symmetry reduction we shall obtain a massive gauge boson, $G_\mu^{\circ'}$, and a massless one, A_μ . These mass eigenstates can be obtain in terms of the gauge eigenstates as follows

$$\begin{aligned} G_\mu^{\circ'} &= \cos \delta_{13} G_\mu^\circ - \sin \delta_{13} B_\mu, \\ A_\mu &= \cos \delta_{13} B_\mu + \sin \delta_{13} G_\mu^\circ, \end{aligned} \quad (2.16)$$

in which the angle δ_{13} is

$$\delta_{13} = \tan^{-1}\left(\sqrt{\frac{2}{3}}\frac{g_1}{g_3}\right). \quad (2.17)$$

Meanwhile the mass of $G_\mu^{\circ'}$ can be obtained as

$$M^2 = \frac{1}{4}(g_1^2 + \frac{3}{2}g_3^2)\phi_0^2, \quad (2.18)$$

where ϕ_0^2 is the vacuum expectation value of ϕ . Although the θ -independent parts of B_μ and G_μ° only present in the Higgsac Lagrangian, we replace (2.16) in the full Lagrangian. However we have to notice that only the θ -independent parts of $G_\mu^{\circ'}$ gets mass through Higgsac mechanism. Hereafter we use G_μ° instead of $G_\mu^{\circ'}$. In this manner, the fermions of the $U_\star(3) \times U_\star(1)$ theory couple to the massless gauge boson of the residual $U_\star(1)$, A_μ , through the usual electric charges if we define

$$\frac{1}{2}g_1 \cos \delta_{13} = e, \quad (2.19)$$

$$-\frac{1}{2\sqrt{6}}g_3 \sin \delta_{13} = q_d, \quad (2.20)$$

where q_d is the electric charge of the down quark, $-\frac{1}{3}e$, and

$$\frac{1}{2}(g_1 \cos \delta_{13} - \frac{1}{\sqrt{6}}g_3 \sin \delta_{13}) = q_u, \quad (2.21)$$

where q_u is the electric charge of the up quark, $\frac{2}{3}e$. Therefore, the QED interactions of fermions in this model are as

$$\mathcal{L}_{e-A_\mu} = -ie\bar{\psi}_e \star \gamma^\mu \psi_e \star A_\mu, \quad (2.22)$$

$$\mathcal{L}_{d-A_\mu} = -\frac{i}{3}e\bar{\psi}_d \star \gamma^\mu \psi_d \star A_\mu, \quad (2.23)$$

$$\mathcal{L}_{u-A_\mu} = \frac{2i}{3}e\bar{\psi}_u \star \gamma^\mu A_\mu \star \psi_u - \frac{1}{3}e\bar{\psi}_u \star \gamma^\mu [\psi_u, A_\mu]_\star, \quad (2.24)$$

for electron, down quark and up quark, respectively. Finally, neutrinos which are the neutral particles can be coupled to photons in the NC space-time through the adjoint representation as

$$\mathcal{L}_{\nu-A_\mu} = -ie\bar{\psi}_\nu \star \gamma^\mu [\psi_\nu, A_\mu]_\star. \quad (2.25)$$

The Feynman rules for this model are completely collected in Appendix B.

3 The Quantization of $U_\star(3) \times U_\star(1)$

The $U_\star(n)$ is a non-Abelian gauge theory for all n , therefore we should perform the gauge fixing to have the nonsingular propagator. The Faddeev-Popov and the gauge fixing terms for this theory are given as [7]

$$S_{GF} = \int d^d x \left(-\frac{1}{2\alpha} \text{Tr}(\partial_\mu G^\mu \partial_\nu G^\nu) + \frac{1}{2} \text{Tr}(i\bar{c} \star \partial_\mu D^\mu c - i\partial_\mu D^\mu c \star \bar{c}) \right), \quad (3.26)$$

where c and \bar{c} are the ghost fields and

$$D_\mu c = \partial_\mu c - ig[G_\mu, c]_\star, \quad (3.27)$$

in which one needs n^2 ghost fields for the n^2 gauge fields. Hence for the $U_\star(3) \times U_\star(1)$ gauge group there are nine ghosts for $U_\star(3)$ and one ghost for $U_\star(1)$ which can be written as

$$\begin{aligned} S_{GF} &= \int d^d x \left(-\frac{1}{2\alpha} \partial_\mu B^\mu \partial_\nu B^\nu + \frac{1}{2} (i\bar{c}_B \star \partial_\mu D^\mu c_B - i\partial_\mu D^\mu c_B \star \bar{c}_B) \right) \\ &+ \int d^d x \left(-\frac{1}{\alpha} \text{Tr}(\partial_\mu G^\mu \partial_\nu G^\nu) + \text{Tr}(i\bar{c} \star \partial_\mu D^\mu c - i\partial_\mu D^\mu c \star \bar{c}) \right), \end{aligned} \quad (3.28)$$

where c_B and c are the ghost fields corresponding to B_μ and G_μ , respectively, and

$$\begin{aligned} D_\mu c_B &= \partial_\mu c_B - ig_1[B_\mu, c_B]_\star, \\ D_\mu c &= \partial_\mu c - ig_3[G_\mu, c]_\star. \end{aligned} \quad (3.29)$$

After the symmetry reduction, as was discussed in the pervious section, the θ -independent part of G_μ° gets mass through the symmetry breaking while the θ -dependent part remains

massless. Therefore it is necessary to take into account a ghost field corresponding to the θ -dependent part of G_μ° which we denote by c° . In the case of $A_\mu(x)$ the θ -independent as well as θ -dependent part remains massless. But the θ -dependent part of $A_\mu(x)$ is still non-Abelian and also needs a ghost field which we denote by c^γ . These new ghost fields can be written in terms of c_B and c_{G° (corresponding to B_μ and G_μ°) as follows

$$\begin{aligned} c^\circ &= \cos \delta_{13} c_{G^\circ} - \sin \delta_{13} c_B, \\ c^\gamma &= \cos \delta_{13} c_B + \sin \delta_{13} c_{G^\circ}. \end{aligned} \quad (3.30)$$

4 The calculation of β -function

A direct one loop calculation in the $U_\star(1)$ gauge theory for both fundamental and adjoint representation have been resulted in a negative β -function [7, 11]. The negative contribution comes from the photon self interaction due to the NC space-time. However, in the $U_\star(3) \times U_\star(1)$ gauge theory the result will be definitely different. In fact the reasons are two folds, the first one is that the charge quantization problem is solved and quarks as well as the leptons are accommodated in the theory. The second one is the appearance of the new interactions among the different gauge bosons i.e. photon and gluon, that makes the result complicated. In this section we explicitly calculate the QED β -function in the $U_\star(3) \times U_\star(1)$ theory. The QED β -function can be obtained by the following relation

$$\beta(e) = M \frac{\partial}{\partial M} (-\delta_1 + e\delta_2 + \frac{e}{2}\delta_3), \quad (4.31)$$

in which δ_1 , δ_2 and δ_3 are the vertex, fermion and photon counter-terms, respectively.

4.1 Electron self energy

The one loop corrections for the electron self energy are shown in Fig.1 in which Fig.1(a) corresponds to the ordinary QED. The UV divergences of these diagrams can be subtracted by the rescaling of the field and mass of the electron. The electron renormalization factor Z_ψ can be easily found as (see appendix A, Eq.(A.41))

$$\begin{aligned} \delta Z_\psi^a &= -\frac{e^2}{16\pi^2} \frac{1}{\epsilon}, \\ \delta Z_\psi^b &= -\frac{e^2}{16\pi^2} \frac{1}{\epsilon} (\tan^2 \delta_{13}), \\ Z_\psi &= 1 - \frac{e^2}{16\pi^2} \frac{1}{\epsilon} (1 + \tan^2 \delta_{13}), \end{aligned} \quad (4.32)$$

where $\frac{1}{\epsilon} = \frac{1}{\epsilon} + \gamma_E - \log 4\pi$ for the space-time dimension $d = 4 - 2\epsilon$. In presenting the explicit expressions of all renormalization constants the MMS scheme is used throughout.

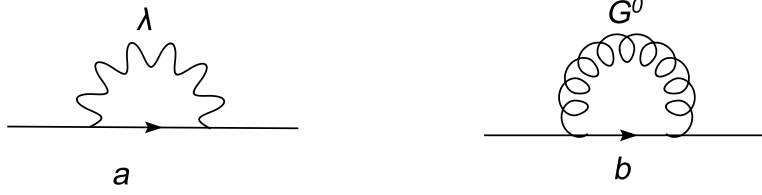


Figure 1: The electron self energy

4.2 Photon self energy

The one loop contributions of the photon self energy are shown in Fig.2. The fermion loop in the Fig.2(a) can be each one of electron, neutrino, up and down quark. The main difference in each loop with respect to the ordinary space is an appearance of a phase factor depends on the NC-parameter. These factors for the charged fermions fortunately cancel each other and therefore the loop-calculations are completely like the ordinary $SU(3) \times U(1)$. The contributions of such diagrams are (see Eq.(A.44)):

$$\begin{aligned}\delta Z_A^{cf} &= -\frac{e^2}{16\pi^2} \frac{1}{\epsilon} \frac{4}{3} (1 + 3((2/9 + 4/9 + 4/9) + 1/9)), \\ &= -\frac{e^2}{16\pi^2} \frac{1}{\epsilon} \frac{4}{3} \left(\frac{14}{3}\right),\end{aligned}\tag{4.33}$$

where cf means charged fermion.

Meanwhile the neutrino loop contribution in the photon self energy is different from the charged one, because the noncommutative phases in this case do not cancel each other. Therefore for such a diagram one has (see Eq.(A.45))

$$\delta Z_A^{nf} = -\frac{e^2}{16\pi^2} \frac{1}{\epsilon} \frac{4}{3} (2),\tag{4.34}$$

where nf means neutral fermion. Then the total contributions from the fermion loops i.e. charged fermions (lepton and quark) and neutrino, are

$$\begin{aligned}\delta Z_A^f &= -\frac{e^2}{16\pi^2} \frac{1}{\epsilon} \frac{4}{3} N_F (1 + 33/9 + 2), \\ &= -\frac{e^2}{16\pi^2} \frac{1}{\epsilon} \frac{4}{3} N_F (20/3),\end{aligned}\tag{4.35}$$

where N_F is the number of generations.

The remaining parts of the Fig.2 show the gauge boson one-loop corrections to the photon self energy. The gauge boson diagrams in Fig.2(b) have singularity proportional to $\frac{1}{q^2}$ that can be canceled by those come from the diagrams given in the Fig.2(c). In fact these diagrams have not any contribution on the β -function of $U_\star(3) \times U_\star(1)$ model [7]. Hence, the one loop corrections for the photon self energy which contain the photon-gluon interactions induced by the NC space-time can be obtained as (see Eq.(A.55))

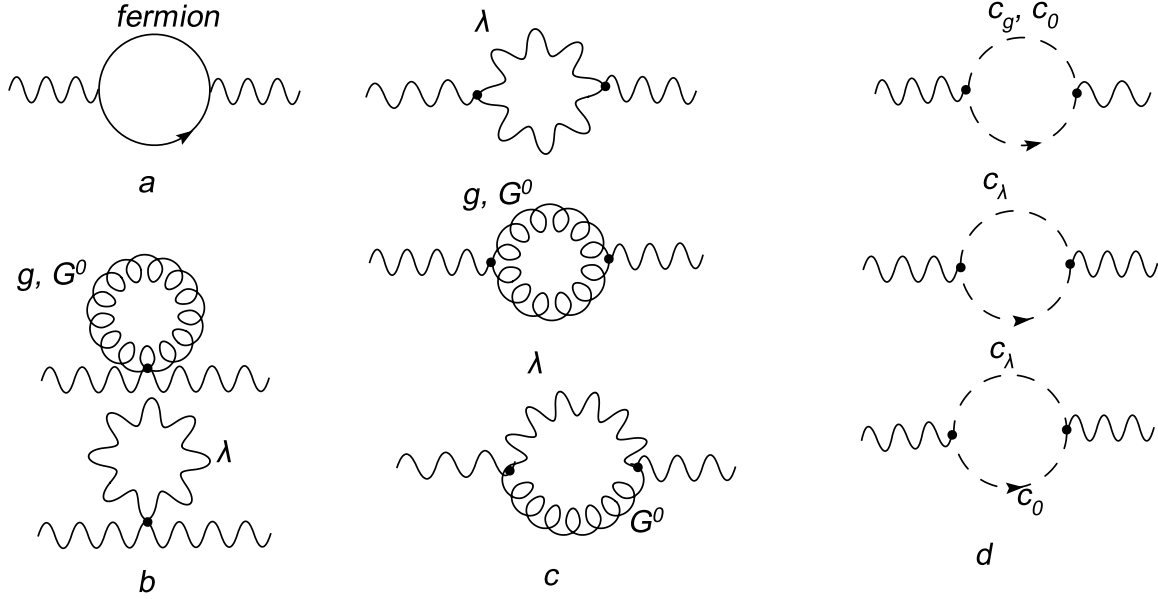


Figure 2: Photon self energy (a) fermion loop (b,c) gauge boson loop (d)ghost boson loop

$$\delta Z_A^g = +\frac{e^2}{16\pi^2} \frac{1}{\epsilon} \frac{10}{3} (1/9)(10 + \delta_{ab}) = +\frac{e^2}{16\pi^2} \frac{1}{\epsilon} \frac{10}{3} (2). \quad (4.36)$$

Therefore the total renormalization constant for the one-loop correction of the photon self energy is

$$\delta Z_A = -\frac{e^2}{16\pi^2} \frac{1}{\epsilon} \frac{20}{3} \left[\frac{4}{3} N_F - 1 \right]. \quad (4.37)$$

4.3 Vertex function

After studying the UV divergences of two point function in the $U_\star(3) \times U_\star(1)$ theory, now we have to calculate the photon-electron vertex function at the one-loop level. There are four diagrams in this case as shown in the Fig.3. The three diagrams in the Fig.3(b) show the NCQED effects in the $U_\star(3) \times U_\star(1)$ theory. It should be noted that here there is two additional diagrams with respect to the NCQED based on the $U_\star(1)$. The QED like diagram given in Fig.3(a) is finite unlike the ordinary QED [7] but for the nonabelian diagrams one can easily find (see Eq.(A.61))

$$\delta Z_{\bar{\psi}\psi A} = -\frac{e^2}{16\pi^2} \frac{1}{\epsilon} 3 \frac{1}{3} (1 + 2 \cos^2 \delta_{13} + \tan^2 \delta_{13} (1 + 2 \sin^2 \delta_{13}) - 2 \tan \delta_{13} \sin 2\delta_{13}). \quad (4.38)$$

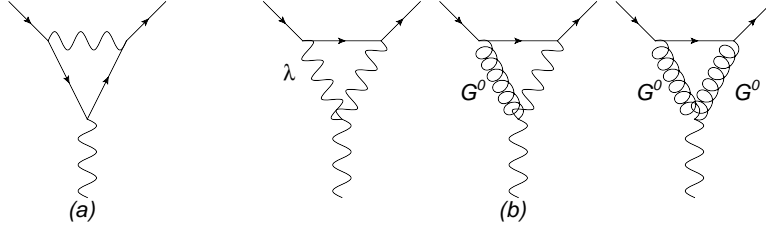


Figure 3: Vertex function (a) *QED* like diagram (b) These diagrams occur through a non abelian vertex

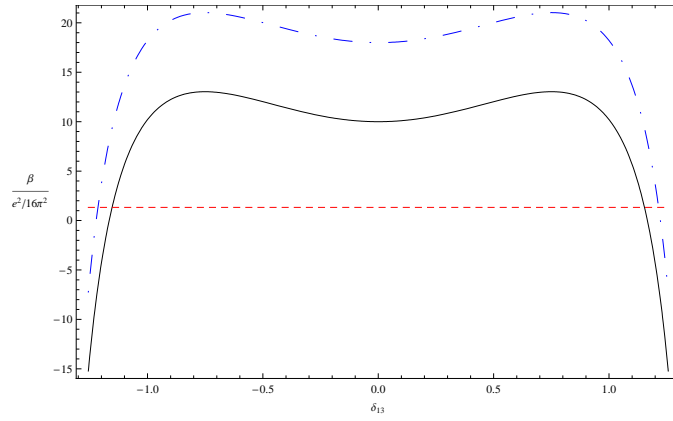


Figure 4: $\frac{16\pi}{e^3}\beta$ versus δ_{13} . Black (solid), blue (dot) and red (dashed) curves represent the β -function corresponding to NCQED without neutrinos, NCQED with neutrinos and usual QED, respectively.

Now the β -function of the NCQED part of the $U_\star(3) \times U_\star(1)$ gauge theory, by using (4.32),(4.37) and (4.38), can be obtained as follows

$$\begin{aligned} \beta(e) = +\frac{e^3}{16\pi^2} \{ & + \frac{4}{3}N_F(1 + 2 + 33/9) + 2(1 + \tan^2 \delta_{13}) \\ & - 2[\frac{5}{3}(2) + (1 + \cos^2 \delta_{13}) \\ & + \tan^2 \delta_{13}(1 + 2 \sin^2 \delta_{13}) - 2 \tan \delta_{13} \sin 2\delta_{13}] \}, \end{aligned} \quad (4.39)$$

where in the first term the contributions of the charged leptons, neutrinos and quarks are given separately. One should note that the neutrino has only contribution to the first term. Meanwhile (4.39) shows that the β -function depends on the mixing parameter, δ_{13} as a free parameter. In fact the β , depending on the value of δ_{13} can be negative, positive or even zero as shown in Fig.4. In the Fig.4 the β -function for the ordinary QED is compared with the NCQED (with and without neutrino). It can be easily seen that the NC QED β -function is equal to the usual QED for $\delta_{13} = 0.367\pi$.

5 Conclusion

We have considered the unexpanded NCQED based on the $U_\star(3) \times U_\star(1)$ gauge group. We have shown that the β function in agreement with [7] does not depend on the parameter of non commutativity but in contrast with the $U_\star(1)$ NCQED it depends on a new parameter i.e. mixing parameter δ_{13} , see (4.39). Therefore the β function can be negative, positive or zero depending on the value of the mixing parameter. Meanwhile the β function in the $U_\star(1)$ NCQED is suffering from the asymptotic freedom that is in contrast with the experiment. We have compared our result with the ordinary QED in the Fig.4 and it is found that for the $\delta_{13} = 0.367\pi$ our β function is equal to the ordinary QED. In fact we have shown that in contrast with the expanded QED the unexpanded NCQED based on the $U_\star(3) \times U_\star(1)$ gauge group is not only one-loop renormalizable but also by fixing δ_{13} it can have the same value for the β function as the commutative QED. Nevertheless the obtained value for δ_{13} needs to confirm in an independent way.

A The calculations of two and three point functions

Electron self energy: The electron self-energy includes two diagrams as are shown in the Fig.1. Using the Feynman rules (B.70) and (B.73) one can write

$$-i\Sigma(p) = (-ie)^2(1 + \tan^2 \delta_{13}) \int \frac{d^4k}{(2\pi)^2} \gamma^\mu \frac{\not{k} + m}{k^2 - m^2} \gamma_\mu \frac{1}{(k - p)^2},$$

$$= (-ie)^2(1 + \tan^2 \delta_{13}) \int_0^1 dx \int \frac{d^d l}{(2\pi)^2} \frac{-2x \not{p} + 4m}{(l^2 - \Delta)^2}, \quad (\text{A.40})$$

where $\Delta = (1 - x)m^2 - x(1 - x)p^2$. Then one has

$$\begin{aligned} \delta Z_\psi &= \frac{d\Sigma}{d\not{p}}|_{\not{p}=m}, \\ &= -\frac{e^2}{16\pi^2}(1 + \tan^2 \delta_{13})\frac{1}{\epsilon}. \end{aligned} \quad (\text{A.41})$$

Photon self energy: All diagrams corresponding to the one loop photon self energy are shown in the Fig.2. First we calculate the fermion loop, Fig.2a, containing electron, u and d quarks as follows

$$i\Pi_{cf}^{\mu\nu} = -(-ie)^2 \int \frac{d^4 k}{(2\pi)^2} (I_e + 3I_u + 3I_d) \frac{\text{Tr}\{\gamma^\mu(\not{k} + m)\gamma^\nu(\not{k} - \not{q} + m)\}}{(k^2 - m^2)((k - q)^2 - m^2)}, \quad (\text{A.42})$$

where the coefficient 3 is a color factor for the quarks u and d . Using the Feynman rules (B.70), (B.71) and (B.72) one can easily find

$$\begin{aligned} I_e &= 1, \\ I_d &= 1/9, \\ I_u &= 4/9 + 2/9 + 8/9 \sin^2(q \times k) = 4/9 + 2/9 + 4/9 + \mathcal{NP}, \end{aligned} \quad (\text{A.43})$$

where \mathcal{NP} means non-planer. In fact the non-planer part has not any contribution to the beta function [7] and it is not needed to calculate this part. Meanwhile the planer parts are similar to the ordinary QED and it can be easily obtained as

$$\begin{aligned} i\Pi_{cf}^{\mu\nu} &= e^2(1 + 33/9) \int_0^1 dx \int \frac{d^d l}{(2\pi)^2} \frac{g^{\mu\nu}(2/d - 1)l^2 - 2x(1 - x)q^\mu q^\nu + g^{\mu\nu}(m^2 + x(1 - x)q^2)}{(l^2 - \Delta)^2}, \\ &= -ie^2(1 + 33/9)(q^2 g^{\mu\nu} - q^\mu q^\nu) \frac{8}{16\pi^2} \int_0^1 x(1 - x) \frac{1}{\epsilon} + \mathcal{NP}, \\ &= -ie^2(1 + 33/9)(q^2 g^{\mu\nu} - q^\mu q^\nu) \frac{1}{16\pi^2} \frac{4}{3} \frac{1}{\epsilon} + \mathcal{NP}, \end{aligned} \quad (\text{A.44})$$

where $\Delta = m^2 - x(1 - x)q^2$. Similarly, the impact of the neutrino loop on the photon self energy, using (B.76), can be easily obtained as follows

$$\begin{aligned} i\Pi_{neutrino}^{\mu\nu} &= -(-ie)^2 \int \frac{d^4 k}{(2\pi)^2} (4 \sin^2(q \times k)) \frac{\text{Tr}\{\gamma^\mu \not{k} \gamma^\nu (\not{k} - \not{q})\}}{(k^2)((k - q)^2)}, \\ &= 2e^2 \int_0^1 dx \int \frac{d^d l}{(2\pi)^2} \frac{g^{\mu\nu}(2/d - 1)l^2 - 2x(1 - x)q^\mu q^\nu + x(1 - x)q^2 g^{\mu\nu}}{(l^2 - \Delta)^2} + \mathcal{NP}, \\ &= -2ie^2(q^2 g^{\mu\nu} - q^\mu q^\nu) \frac{1}{16\pi^2} \frac{4}{3} \frac{1}{\epsilon} + \mathcal{NP}, \end{aligned} \quad (\text{A.45})$$

where $\Delta = -x(1-x)q^2$. Therefore the total contribution of the fermion loops can be obtained as

$$\delta Z_A^f = -\frac{e^2}{16\pi^2} \frac{4}{3} N_F (1 + 2 + 33/9). \quad (\text{A.46})$$

Now we consider the contribution of the gauge bosons on the self-energy of photon, Fig.2b and Fig.2c. Let us start with the diagrams given in Fig.2b. Using the Feynman rules (B.95) and (B.97), the corresponding self energy can be found as follows

$$\begin{aligned} i\Pi_{Fig.2b}^{\mu\nu} &= -\frac{1}{2}e^2(2/3)^2(10 + \delta_{ab}) \int \frac{d^4k}{(2\pi)^2} 2(d-1)g^{\mu\nu} \sin^2(q \times k) \frac{1}{k^2}, \\ &= -\frac{1}{2}e^2(2/3)^2(10 + \delta_{ab}) \int \frac{d^4k}{(2\pi)^2} (d-1)g^{\mu\nu} \frac{1}{k^2} \frac{(q+p)^2}{(q+p)^2} + \mathcal{NP}, \end{aligned} \quad (\text{A.47})$$

or for the planar part

$$i\Pi_{Fig.2b}^{\mu\nu} = -e^2 \frac{1}{2} (2/3)^2 (10 + \delta_{ab}) \int_0^1 dx \int \frac{d^d l}{(2\pi)^2} \frac{(d-1)g^{\mu\nu}(l^2 + x(1-x)q^2)}{(l^2 - \Delta)^2}, \quad (\text{A.48})$$

where $\Delta = -x(1-x)p^2$, $1/2$ is a symmetry factor and the dimensional regularization is used throughout. Then after performing the integral on the momenta we have

$$\begin{aligned} i\Pi_{Fig.2b}^{\mu\nu} &= \frac{ie^2}{16\pi^2} \frac{1}{2} (2/3)^2 (10 + \delta_{ab}) \int_0^1 dx \frac{q^2 g^{\mu\nu}}{\Delta^{2-d/2}} \\ &\quad (-\Gamma(1-d/2)d/2(d-1)x(1-x) - \Gamma(2-d/2)(d-1)(1-x)^2). \end{aligned} \quad (\text{A.49})$$

The contribution of the diagrams given in the Fig.2c on the photon self energy can be easily obtained in a similar way as follows

$$\begin{aligned} i\Pi_{Fig.2c}^{\mu\nu} &= -\frac{1}{2}e^2(2/3)^2(10 + \delta_{ab}) \int \frac{d^4k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(q+k)^2} \\ &\quad [-g^{\mu\nu}[(2k+q)^2 + (q-p)^2] - d(k+2q)^\mu(k+2q)^\nu + [(2k+q)^\mu(k+2q)^\nu \\ &\quad + (k-q)^\mu(2k+q)^\nu - (k+2q)^\mu(k-q)^\nu + (\mu \leftrightarrow \nu)], \end{aligned} \quad (\text{A.50})$$

or

$$\begin{aligned} i\Pi_{Fig.2c}^{\mu\nu} &= -\frac{1}{2}e^2(2/3)^2(10 + \delta_{ab}) \int_0^1 dx \int \frac{d^d l}{(2\pi)^2} \frac{1}{(l^2 - \Delta)^2} [-g^{\mu\nu}(l^2 6(1-1/d) \\ &\quad - q^2(3-2x+2x^2)) + q^\mu q^\nu [(2-d)(1-2x)^2 + 2(1+x)(2-x)]], \end{aligned} \quad (\text{A.51})$$

and after performing the integral on the momentum the self energy leads to

$$\begin{aligned} i\Pi_{Fig.2c}^{\mu\nu} &= \frac{ie^2}{16\pi^2} \frac{1}{2} (2/3)^2 (10 + \delta_{ab}) \int_0^1 dx \frac{q^2 g^{\mu\nu}}{\Delta^{2-d/2}} \\ &\quad [\Gamma(1-d/2)g^{\mu\nu}q^2[3/2(d-1)(x-x^2)] + \Gamma(2-d/2)g^{\mu\nu}q^2(1/2)[3-2x+2x^2] \\ &\quad - \Gamma(2-d/2)q^\mu q^\nu(1/2)[(2-d)(1-2x)^2 + 2(1+x)(2-x)]]. \end{aligned} \quad (\text{A.52})$$

Here, one should note that for the two photons (using (B.84)), two \hat{G}^0 (using (B.89)) and two gluons (B.86) in the loop the symmetry factor is 1/2 but for the photon- \hat{G}^0 (using (B.88)) in the loop the factor is one. Finally, we must include the diagrams containing ghost loops (Fig.2d). The corresponding Feynman rules are photon-ghost of photon (B.103); photon-ghost of G_μ° (B.104); photon-ghost of gluons (B.109); photon-ghost of photon and G_μ° (B.107). Therefore for the corresponding self energy one has

$$i\Pi_{Fig.2d}^{\mu\nu} = -e^2(2/3)^2(10 + \delta_{ab}) \int \frac{d^4k}{(2\pi)^2} \sin^2(q \times k) \frac{i(k+q)^\mu}{k^2} \frac{ik^\nu}{(q+k)^2}, \quad (\text{A.53})$$

or after manipulating some algebra one finds

$$\begin{aligned} i\Pi_{Fig.2d}^{\mu\nu} &= \frac{ie^2}{16\pi^2} \frac{1}{2} (2/3)^2 (10 + \delta_{ab}) \int_0^1 dx \frac{q^2 g^{\mu\nu}}{\Delta^{2-d/2}} \\ &\quad (-\Gamma(1-d/2) g^{\mu\nu} q^2 [x(1-x)/2] + \Gamma(2-d/2) q^\mu q^\nu [x(1-x)]), \end{aligned} \quad (\text{A.54})$$

which leads to

$$\begin{aligned} i\Pi_{gauge}^{\mu\nu} &= i\Pi_{Fig.2d}^{\mu\nu} + i\Pi_{Fig.2c}^{\mu\nu} + i\Pi_{Fig.2b}^{\mu\nu}, \\ &= i(q^2 g^{\mu\nu} - q^\mu q^\nu) \frac{10}{3} (1/9) (10 + \delta_{ab}) \frac{e^2}{16\pi^2} \frac{1}{\epsilon} + \dots, \end{aligned} \quad (\text{A.55})$$

and therefore

$$\delta Z_A = -\frac{e^2}{16\pi^2} \frac{1}{\epsilon} \left[\frac{4}{3} N_F (1 + 2 + 33/9) - \frac{10}{3} (1/9) (10 + \delta_{ab}) \right]. \quad (\text{A.56})$$

Now we consider the vertex function and the corresponding diagrams which are given in the Fig.3. The diagram given in the Fig.3a is very similar to its counterpart in the ordinary *QED* but with different divergence behavior. Here using the NC rules one can easily find

$$\begin{aligned} -ie\Gamma^\mu &= -ie \exp(ip \times \hat{p}) (\gamma^\mu + \delta\Gamma^\mu), \\ \delta\Gamma_{Fig.3a}^\mu &= -ie^2 (1 + \tan^2 \delta_{13}) \int \frac{d^4k}{(2\pi)^4} \frac{\exp(iq \times k) \gamma^\lambda (\not{k} + \not{q} + m) \gamma^\mu (\not{k} + m) \gamma_\lambda}{(k^2 - m^2)((k+q)^2 - m^2)(p-k)^2}, \end{aligned} \quad (\text{A.57})$$

where the factor $\exp(iq \times k)$ causes the equation to be finite. Meanwhile for the diagrams given in Fig.3b we have

$$\begin{aligned} \delta\Gamma_{Fig.3b}^\mu &= +(2/3)e^2 I_v \int \frac{d^4k}{(2\pi)^4} \frac{2i \sin^2(q \times k) + 2 \cos(q \times k) \sin(q \times k)}{k^2 (k+q)^2 ((p-k)^2 - m^2)} \\ &\quad \times (\gamma^\lambda (\partial - \not{k} + m) \gamma^\rho) [-(2k+q)^\mu g^{\rho\lambda} + (2q+k)^\rho g^{\mu\lambda} + (k-q)^\lambda g^{\mu\rho}], \end{aligned} \quad (\text{A.58})$$

or

$$\delta\Gamma_{Fig.3b}^\mu = +i(2/3)e^2 I_v \int \frac{d^4k}{(2\pi)^4} \frac{-4\not{k}\not{k}^\mu - 2k^2 \gamma^\mu}{k^2 (k+q)^2 ((p-k)^2 - m^2)} + \dots, \quad (\text{A.59})$$

where

$$I_v = 1 + 2 \cos^2 \delta_{13} + (1 + 2 \sin^2 \delta_{13}) \tan^2 \delta_{13} - 2 \tan \delta_{13} (\sin 2\delta_{13}), \quad (\text{A.60})$$

and ... means the finite part of the integral. Therefore $\delta Z_{\bar{\psi} A \psi}$ can be easily found as follows

$$\delta Z_{\bar{\psi} A \psi} = -\frac{e^2}{16\pi^2} 3(1/3)(1 + 2 \cos^2 \delta_{13} + (1 + 2 \sin^2 \delta_{13}) \tan^2 \delta_{13} - 2 \tan \delta_{13} (\sin 2\delta_{13})) \frac{1}{\epsilon}. \quad (\text{A.61})$$

B Feynman rules for $NC(SU(3) \times U(1))$ model.

Besides the fermion part of the action that is given in section 2, after symmetry reduction, one can obtain the action of the gauge interactions in terms of the mass eigenstates that is given in the first part of this appendix. In the second part we give the Feynman rules for $NC(SU(3) \times U(1))$ model. Here the fields are physical and for simplicity G_μ° is used instead of $G_\mu^{0'}$ as follows

$$S_{photon} = -\frac{1}{4} \int \{ \partial_{[\mu} A_{\nu]} \star \partial^{[\mu} A^{\nu]} + \frac{4}{3} i e (1 + 2 \cos^2 \delta) \partial_{[\mu} A_{\nu]} \star [A^\mu, A^\nu]_\star - \frac{4}{9} e^2 (1 + 8 \cos^2 \delta_{13}) [A_\mu, A_\nu]_\star \star [A^\mu, A^\nu]_\star \} d^4 x, \quad (\text{B.62})$$

$$S_{G_\mu^\circ} = -\frac{1}{4} \int \{ \partial_{[\mu} G_{\nu]}^\circ \star \partial^{[\mu} G^{\circ \nu]} - 2 i e (2 \tan \delta_{13} \sin^2 \delta_{13} - \frac{2}{3} \cot \delta_{13} \cos^2 \delta_{13}) \partial_{[\mu} G_{\nu]}^\circ \star [G^{\circ \mu}, G^{\circ \nu}]_\star - 4 e^2 (\tan^2 \delta_{13} \sin^2 \delta_{13} + \frac{1}{9} \cot^2 \delta_{13} \cos^2 \delta_{13}) [G_\mu^\circ, G_\nu^\circ]_\star \star [G^{\circ \mu}, G^{\circ \nu}]_\star \} d^4 x, \quad (\text{B.63})$$

$$S_{photon-G_\mu^\circ} = -\frac{1}{4} \int \{ -\frac{4}{3} i e \sin 2\delta_{13} \left(\partial_{[\mu} A_{\nu]} \star ([A^\mu, G^{\circ \nu}]_\star + [G^{\circ \mu}, A^\nu]_\star) + \partial_{[\mu} G_{\nu]}^\circ \star [A^\mu, A^\nu]_\star \right) + 4 i e (1 - \frac{2}{3} \cos^2 \delta_{13}) \left(\partial_{[\mu} G_{\nu]}^\circ \star ([A^\mu, G^{\circ \nu}]_\star + [G^{\circ \mu}, A^\nu]_\star) + \partial_{[\mu} A_{\nu]} \star [G^{\circ \mu}, G^{\circ \nu}]_\star \right) + \frac{16}{9} e^2 \sin 2\delta_{13} [A_\mu, A_\nu]_\star \star ([A^\mu, G^{\circ \nu}]_\star + [G^{\circ \mu}, A^\nu]_\star) + 4 e^2 (\tan \delta_{13} \sin^2 \delta_{13} - \cot \delta_{13} \cos^2 \delta_{13}) [G_\mu^\circ, G_\nu^\circ]_\star \star ([G^{\circ \mu}, A^\nu]_\star + [A^\mu, G^{\circ \nu}]_\star) - 8 e^2 (1 - \frac{8}{9} \cos^2 \delta_{13}) [A_\mu, A_\nu]_\star \star [G^{\circ \mu}, G^{\circ \nu}]_\star \} d^4 x, \quad (\text{B.64})$$

$$S_{gluon} = -\frac{1}{4} \int \{ \partial_{[\mu} G_{\nu]}^a \star \partial^{[\mu} G_a^{\nu]} + 2 i \sqrt{\frac{2}{3 \sin \delta_{13}}} \frac{e}{3} (i f_{abc} \partial_{[\mu} G_{\nu]}^c \star \{G^{a\mu}, G^{b\nu}\}_\star + d_{abc} \partial_{[\mu} G_{\nu]}^c \star [G^{a\mu}, G^{b\nu}]_\star) - \frac{6}{9} \frac{e^2}{\sin^2 \delta_{13}} \left(-f_{abc} f^{dec} \{G_\mu^a, G_\nu^b\}_\star \star \{G_d^\mu, G_e^\nu\}_\star + 2 i f_{abc} d^{dec} \{G_\mu^a, G_\nu^b\}_\star \star [G_d^\mu, G_e^\nu]_\star + d_{abC} d^{deC} [G_\mu^a, G_\nu^b]_\star \star [G_d^\mu, G_e^\nu]_\star \right) \} d^4 x, \quad (\text{B.65})$$

$$\begin{aligned}
S_{gluon-photon} = & -\frac{1}{4} \int \left\{ 2i\sqrt{\frac{2}{3}} e d_{ab0} \partial_{[\mu} A_{\nu]} \star [G_{\mu}^a, G_{\nu}^b]_{\star} + 4ie\sqrt{\frac{2}{3}} \partial_{[\mu} G_{\nu]}^b \star ([G_b^{\mu}, A^{\nu}]_{\star} + [A^{\mu}, G_b^{\nu}]_{\star}) \right. \\
& - \frac{8}{9} e^2 \delta_{ab} [A_{\mu}, A_{\nu}]_{\star} \star [G^{a\mu}, G^{b\nu}]_{\star} \\
& \left. - \frac{8e^2}{3 \sin \delta_{13}} (i f_{abc} \{G_{\mu}^a, G_{\nu}^b\}_{\star} + d_{abc} [G_{\mu}^a, G_{\nu}^b]_{\star}) \star ([G^{c\mu}, A^{\nu}]_{\star} + [A^{\mu}, G^{c\nu}]_{\star}) \right\} d^4x,
\end{aligned} \tag{B.66}$$

$$\begin{aligned}
S_{gluon-G_{\mu}^{\circ}} = & -\frac{1}{4} \int \left\{ 2i\sqrt{\frac{2}{3}} e \cot \delta_{13} d_{ab0} \partial_{[\mu} G_{\nu]}^{\circ} \star [G_{\mu}^a, G_{\nu}^b]_{\star} \right. \\
& + 4ie\sqrt{\frac{2}{3}} \cot \delta_{13} \partial_{[\mu} G_{\nu]}^b \star ([G_b^{\mu}, G^{\circ\nu}]_{\star} + [G^{\circ\mu}, G_b^{\nu}]_{\star}) \\
& - \frac{8}{9} e^2 \cot^2 \delta_{13} \delta_{ab} [G_{\mu}^{\circ}, G_{\nu}^{\circ}]_{\star} \star [G^{a\mu}, G^{b\nu}]_{\star} \\
& - \frac{8e^2 \cot \delta_{13}}{3 \sin \delta_{13}} (i f_{abc} \{G_{\mu}^a, G_{\nu}^b\}_{\star} + d_{abc} [G_{\mu}^a, G_{\nu}^b]_{\star}) \star ([G^{\circ\mu}, G^{c\nu}]_{\star} \\
& \left. + [G^{c\mu}, G^{\circ\nu}]_{\star}) \right\} d^4x,
\end{aligned} \tag{B.67}$$

$$S_{gluon-G_{\mu}^{\circ}-photon} = -\frac{1}{4} \int \left\{ -\frac{8}{9} e^2 \cot \delta_{13} \delta_{ab} [G_{\mu}^a, G_{\nu}^b]_{\star} \star ([A^{\mu}, G^{\circ\nu}]_{\star} + [G^{\circ\mu}, A^{\nu}]_{\star}) \right\} d^4x, \tag{B.68}$$

$$\begin{aligned}
S_{ghosts-gauges} = & \int \left\{ e(1 - \frac{2}{3} \sin^2 \delta_{13}) \bar{c}^{\gamma} \star \partial^{\mu} [A_{\mu}, c^{\gamma}]_{\star} - \frac{e}{3} \sin 2\delta_{13} \bar{c}^{\gamma} \star \partial^{\mu} [G_{\mu}^{\circ}, c^{\gamma}]_{\star} \right. \\
& + (-e \tan \delta_{13} \sin^2 \delta_{13} + \frac{e}{3} \cot \delta_{13} \cos^2 \delta_{13}) \bar{c}^{\circ} \star \partial^{\mu} [G_{\mu}^{\circ}, c^{\circ}]_{\star} \\
& + e(1 - \frac{2}{3} \cos^2 \delta_{13}) (\bar{c}^{\circ} \star \partial^{\mu} [A_{\mu}, c^{\circ}]_{\star} + \bar{c}^{\circ} \star \partial^{\mu} [G_{\mu}^{\circ}, c^{\gamma}]_{\star} + \bar{c}^{\gamma} \star \partial^{\mu} [G_{\mu}^{\circ}, c^{\circ}]_{\star}) \\
& - \frac{e}{3} \sin 2\delta (\bar{c}^{\circ} \star \partial^{\mu} [A_{\mu}, c^{\gamma}]_{\star} + \bar{c}^{\gamma} \star \partial^{\mu} [A_{\mu}, c^{\circ}]_{\star}) + \frac{e}{3} \cot \delta_{13} \bar{c}_b \star \partial^{\mu} [G_{\mu}^{\circ}, c^b]_{\star} \\
& + \frac{e}{3} \bar{c}_b \star \partial^{\mu} [A_{\mu}, c^b]_{\star} + \frac{e}{3} \bar{c}_b \star \partial^{\mu} [G_{\mu}^b, c^{\gamma}]_{\star} + \frac{e}{3} \bar{c}^{\gamma} \star \partial^{\mu} [G_{\mu}^b, c_b]_{\star} \\
& + \frac{e}{3} \cot \delta_{13} \bar{c}_b \star \partial^{\mu} [G_{\mu}^b, c^{\circ}]_{\star} + \frac{e}{3} \cot \delta_{13} \bar{c}^{\circ} \star \partial^{\mu} [G_{\mu}^b, c_b]_{\star} \\
& \left. + i g_3 f_{abc} \bar{c}^c \star \partial^{\mu} \{G_{\mu}^a, c^b\}_{\star} + g_3 d_{abc} \bar{c}^c \star \partial^{\mu} [G_{\mu}^a, c^b]_{\star} \right\} d^4x.
\end{aligned} \tag{B.69}$$

Gluons carry Lorentz indices μ, ν, \dots , color indices a, b, \dots , and momenta p, q, \dots . Ghosts carry only the last two type of labels. All the momenta are entering unless otherwise specified.

electron-photon vertex:

$$-ie \exp(ip \times k) \gamma^{\mu}, \tag{B.70}$$

down quark–photon vertex:

$$-\frac{1}{3}ie \exp(ip \times k) \gamma^\mu, \quad (\text{B.71})$$

up quark–photon vertex:

$$\frac{2}{3}ie(\cos(ip \times k) + 2i \sin(p \times q)) \gamma^\mu, \quad (\text{B.72})$$

electron– G_μ° vertex:

$$ie \tan \delta_{13} \exp(ip \times k) \gamma^\mu, \quad (\text{B.73})$$

down quark– G_μ° vertex:

$$-\frac{1}{3}ie \cot \delta_{13} \exp(ip \times k) \gamma^\mu, \quad (\text{B.74})$$

up quark– G_μ° vertex:

$$-ie\{(\tan \delta_{13} - \frac{1}{3} \cot \delta_{13}) \exp(ik \times p) - \frac{2i}{3} \cot \delta_{13} \sin(k \times p)\} \gamma^\mu, \quad (\text{B.75})$$

neutrino–photon vertex:

$$2e \sin(ip \times k) \gamma^\mu, \quad (\text{B.76})$$

neutrino– G_μ° vertex:

$$-2e \tan \delta_{13} \sin(ip \times k) \gamma^\mu, \quad (\text{B.77})$$

up and down quark–gluon vertex:

$$-\frac{1}{2}ig_3 \exp(ip \times k) \gamma^\mu T^a, \quad (\text{B.78})$$

where p and k are the matter field and gauge field, respectively. Also we have defined $p \times k = \frac{1}{2} \theta^{\mu\nu} p_\mu k_\nu$.

gluon propagator.

$$-\frac{i}{p^2} \delta_{ab} g_{\mu\nu} \quad (\text{B.79})$$

photon and G_μ° propagator.

$$-\frac{i}{p^2} g_{\mu\nu} \quad (\text{B.80})$$

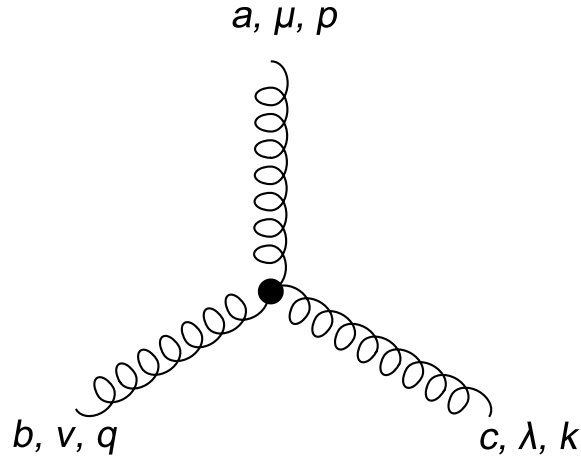


Figure 5: 3-gluon

ghost(photon and G_μ°) propagator.

$$-\frac{i}{p^2}g_{\mu\nu} \quad (\text{B.81})$$

ghost(gluon) propagator.

$$-\frac{i}{p^2}\delta_{ab}g_{\mu\nu} \quad (\text{B.82})$$

3-gluon vertex.

$$-g_3\sin\delta_{13}(f_{abc}\cos(p\times q)+d_{abc}\sin(p\times q))I \quad (\text{B.83})$$

3-photon vertex.

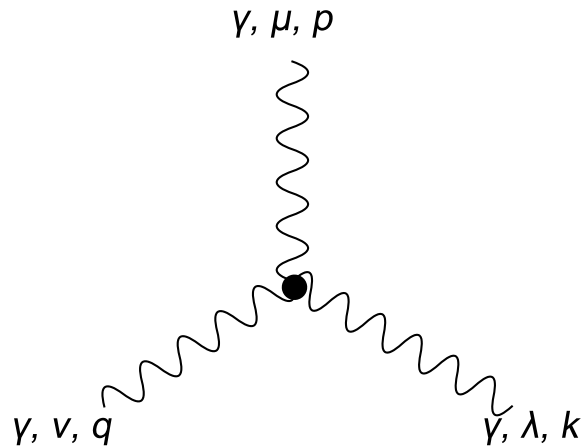


Figure 6: 3-photon

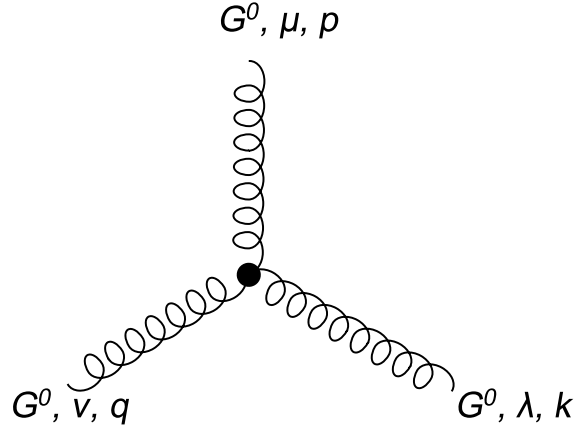


Figure 7: $3\text{-}G^\circ$

$$-\frac{2}{3}e(1 + 2 \cos^2 \delta_{13}) \sin(p \times q)I \quad (\text{B.84})$$

$3\text{-}G^\circ$ vertex.

$$e(2 \tan \delta_{13} \sin^2 \delta_{13} - \frac{2}{3} \cot \delta_{13} \cos^2 \delta_{13}) \sin(p \times q)I \quad (\text{B.85})$$

photon-two gluons vertex.

$$-e\frac{2}{3}\delta_{ab} \sin(p \times q)I \quad (\text{B.86})$$

G° -two gluons vertex.

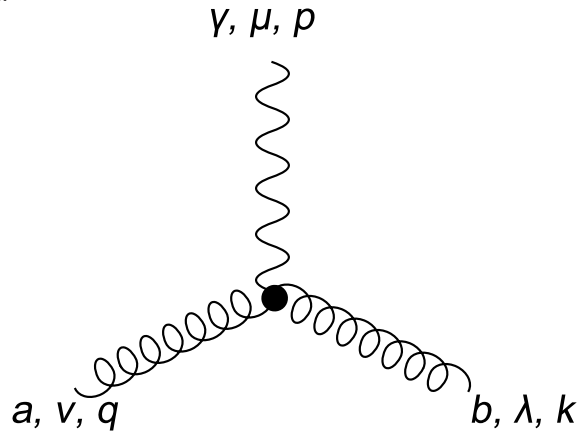


Figure 8: photon - gluon

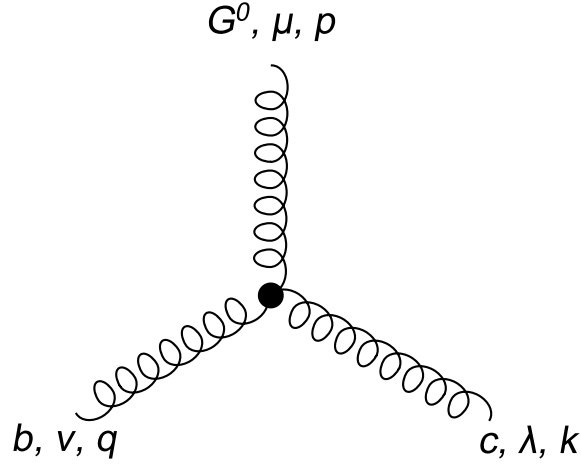


Figure 9: G° - gluon

$$-e\frac{2}{3}\delta_{ab}\cot\delta_{13}\sin(p\times q)I \quad (\text{B.87})$$

G° -two photons vertex.

$$-\frac{2}{3}e\sin 2\delta_{13}\sin(p\times q)I \quad (\text{B.88})$$

photon-two G° vertex.

$$-\frac{2}{3}e(1+2\sin^2\delta_{13})\sin(p\times q)I \quad (\text{B.89})$$

Where I is :

$$I = (g_{\mu\nu}(p-q)_\lambda + g_{\nu\lambda}(q-k)_\mu + g_{\lambda\mu}(k-p)) \quad (\text{B.90})$$

photon-three gluons vertex.

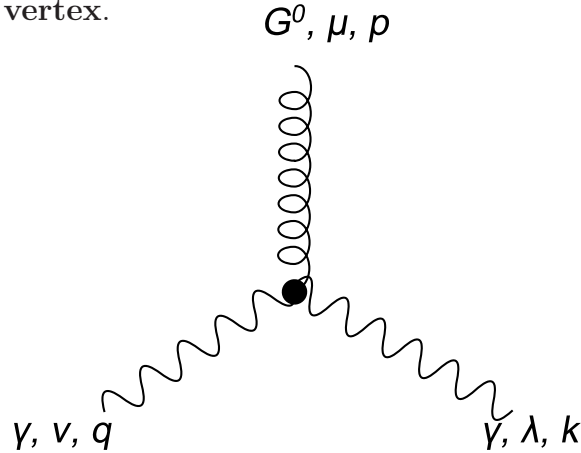


Figure 10: G° - two photons

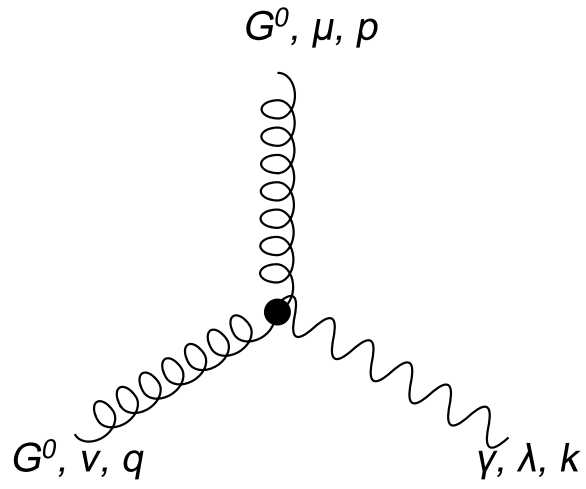


Figure 11: two G^0 - photon

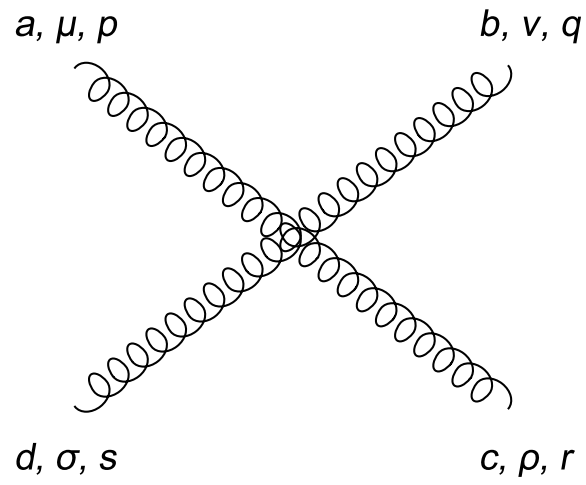


Figure 12: G^0 - gluon

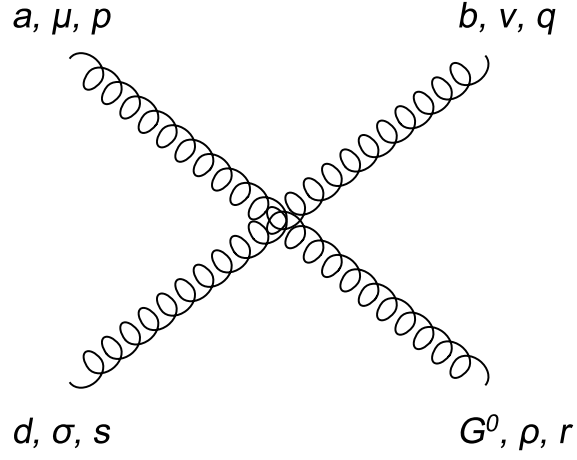


Figure 13: G° - three gluons (photon - three gluons)

$$-\frac{8ie^2}{3\sin\delta_{13}}I_1 \quad (\text{B.91})$$

G° –three gluons vertex.

$$-\frac{8ie^2\cot\delta_{13}}{3\sin\delta_{13}}I_1 \quad (\text{B.92})$$

two G° –two gluons vertex.

$$-2i\left(\frac{2e}{3}\right)^2\delta_{ab}\cot^2\delta_{13}I_2 \quad (\text{B.93})$$

two photon–two gluons vertex.

$$-2i\left(\frac{2e}{3}\right)^2\delta_{ab}I_2 \quad (\text{B.94})$$

4– photon vertex.

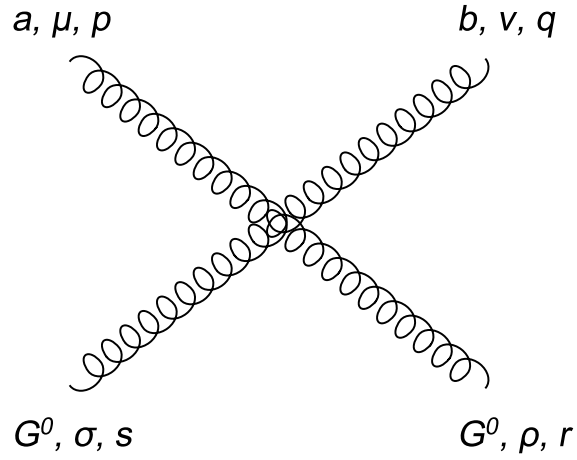


Figure 14: two G° - two gluon

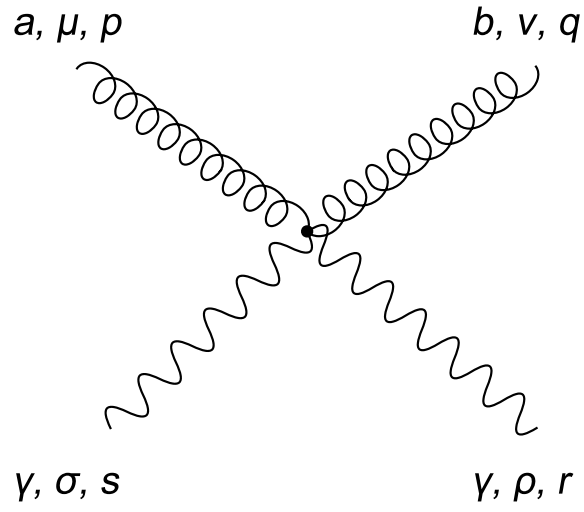


Figure 15: two photon - two gluon

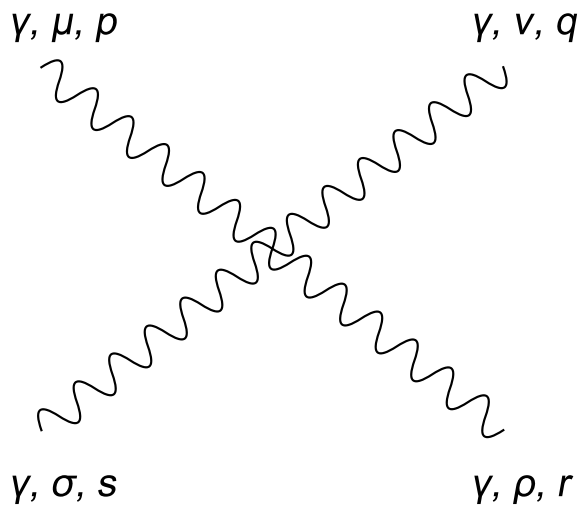


Figure 16: 4- photon

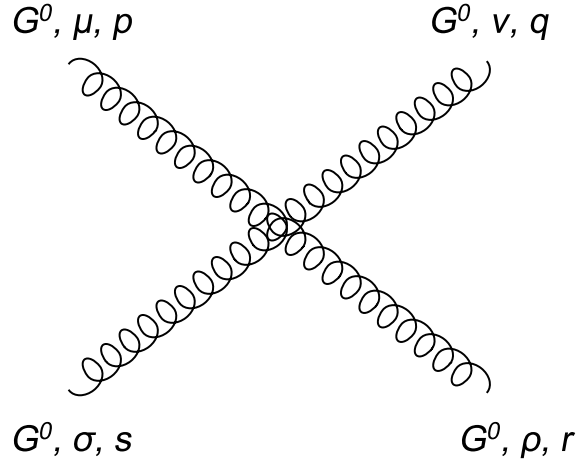


Figure 17: 4- G°

$$-i\left(\frac{2e}{3}\right)^2(1 + 8 \cos^2 \delta_{13})I_2 \quad (\text{B.95})$$

4- G° vertex.

$$-4ie^2(1/9 \cot^2 \delta_{13} \cos^2 \delta_{13} + \tan^2 \delta_{13} \sin^2 \delta_{13})I_2 \quad (\text{B.96})$$

two photons-two G° vertex.

$$-i\left(\frac{2e}{3}\right)^2(1 + 8 \sin^2 \delta_{13})I_2 \quad (\text{B.97})$$

three photons- G° vertex.

$$i\left(\frac{4e}{3}\right)^2 \sin 2\delta_{13}I_2 \quad (\text{B.98})$$

photon- G° -two gluons vertex.

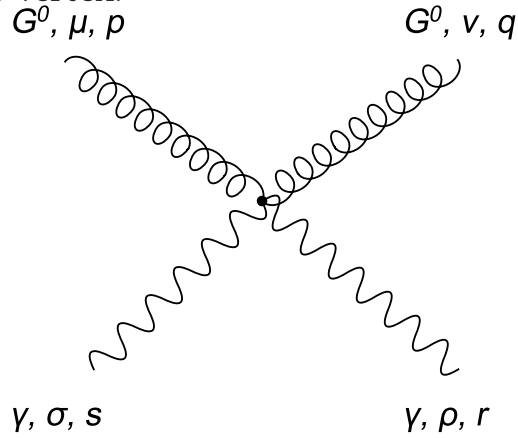


Figure 18: two G° - two photon

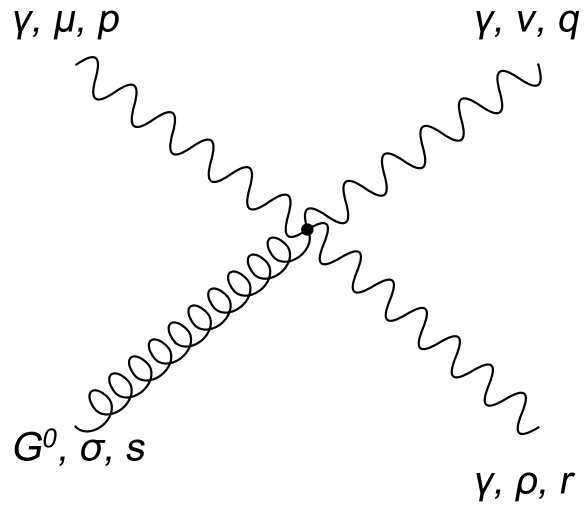


Figure 19: G^0 - three photon

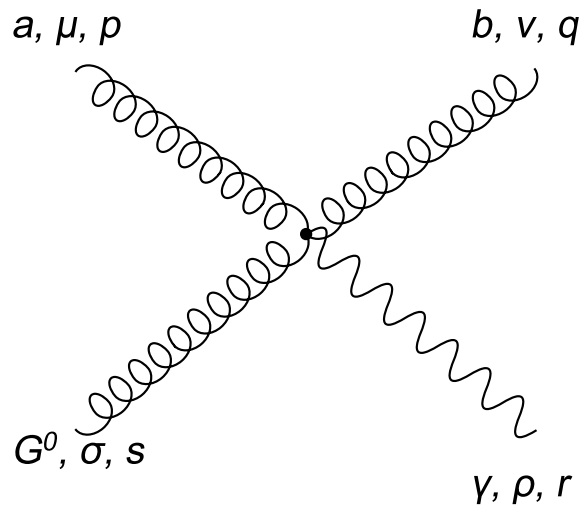


Figure 20: G^0 - photon - two gluon

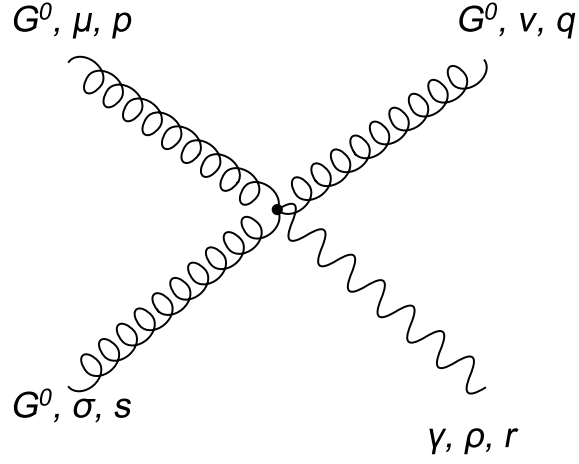


Figure 21: three G° - photon

$$-i\frac{8}{9}e^2 \cot \delta_{13} \delta_{ab} I_2 \quad (\text{B.99})$$

photon–three G° vertex.

$$-4ie^2(\cot \delta_{13} \cos^2 \delta_{13} - \tan \delta_{13} \sin^2 \delta_{13}) I_2 \quad (\text{B.100})$$

Where I_1, I_2 are

$$\begin{aligned} I_1 = \sqrt{\frac{2}{3}} [& (f_{abc} \cos(p \times q) + d_{abc} \sin(p \times q)) \\ & \cdot \sin(r \times s)(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) \\ & + (f_{acb} \cos(s \times q) + d_{acb} \sin(s \times q)) \\ & \cdot \sin(p \times r)(g_{\mu\sigma} g_{\nu\rho} - g_{\mu\nu} g_{\rho\sigma}) \\ & + (f_{acb} \cos(p \times s) + d_{acb} \sin(p \times s)) \\ & \cdot \sin(q \times r)(g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma})] \end{aligned} \quad (\text{B.101})$$

$$\begin{aligned} I_2 = [& \sin(p \times q) \sin(r \times s)(g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) \\ & + \sin(p \times r) \sin(s \times q)(g_{\mu\sigma} g_{\nu\rho} - g_{\mu\nu} g_{\rho\sigma}) \\ & + \sin(p \times s) \sin(q \times r)(g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma})] \end{aligned} \quad (\text{B.102})$$

photon–ghosts(photon) vertex

$$\frac{2}{3}e(1 + 2 \cos^2 \delta_{13}) p_\mu \sin(p \times q) \quad (\text{B.103})$$

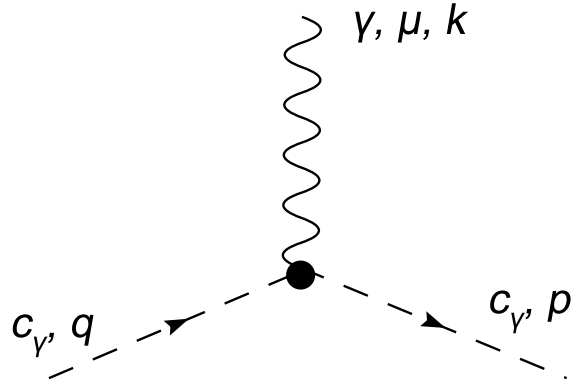


Figure 22: photon-ghosts(photon)

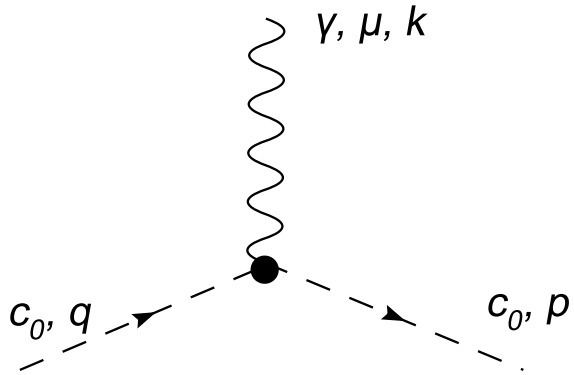


Figure 23: photon-ghosts(G°)

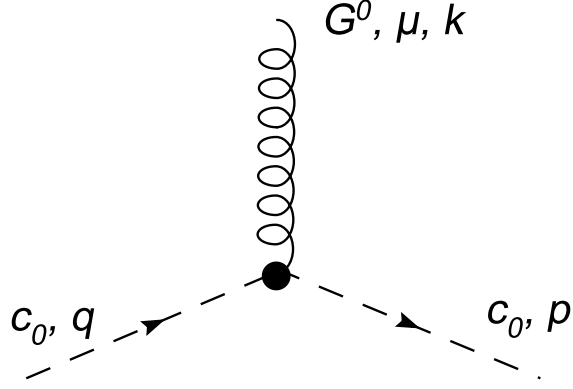


Figure 24: G° -ghosts(G°)

photon-ghosts(G°) vertex

$$\frac{2}{3}e(1 + 2\sin^2 \delta_{13}) p_\mu \sin(p \times q) \quad (\text{B.104})$$

G° -ghosts(G°) vertex

$$-e\left(\frac{1}{3}\cot \delta_{13}\cos^2 \delta_{13} - \tan \delta_{13}\sin^2 \delta_{13}\right) p_\mu \sin(p \times q) \quad (\text{B.105})$$

G° -ghosts(photon) vertex.

$$-\frac{2}{3}e\sin 2\delta_{13} p_\mu \sin(p \times q) \quad (\text{B.106})$$

photon-ghosts(photon- G°) vertex

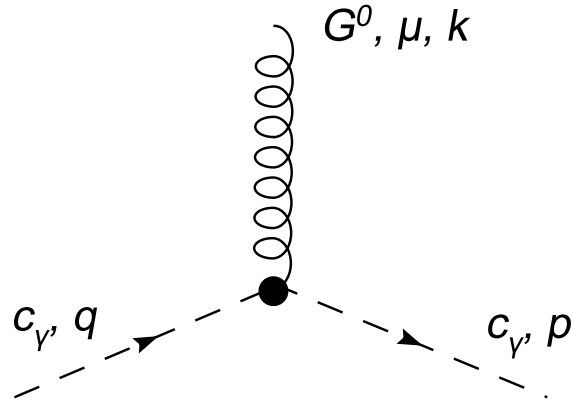


Figure 25: G° -ghosts(photon)

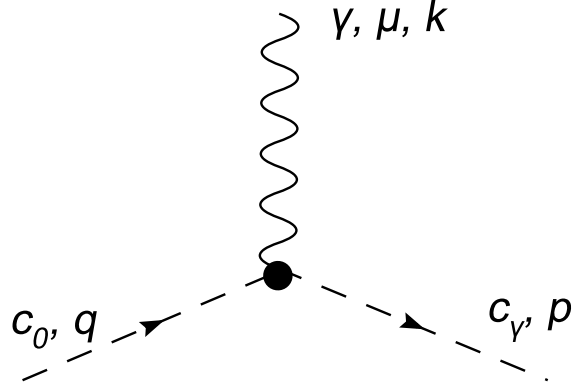


Figure 26: photon-ghosts(photon - G°)

$$-\frac{2}{3}e \sin 2\delta_{13} p_\mu \sin(p \times q) \quad (\text{B.107})$$

G° -ghosts(photon- G°) vertex

$$\frac{2}{3}e(1 + 2\sin^2 \delta_{13}) p_\mu \sin(p \times q) \quad (\text{B.108})$$

photon-ghosts(gluon) vertex

$$\frac{2}{3}e\delta_{ab} p_\mu \sin(p \times q) \quad (\text{B.109})$$

G° -ghosts(gluon) vertex

$$-\frac{2}{3}e\delta_{ab} \cot \delta_{13} p_\mu \sin(p \times q) \quad (\text{B.110})$$

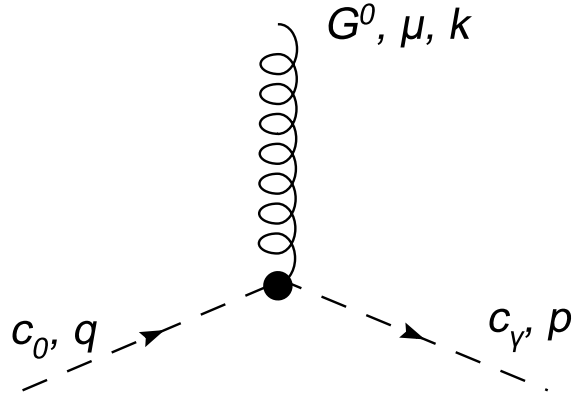


Figure 27: G° -ghosts(photon - G°)

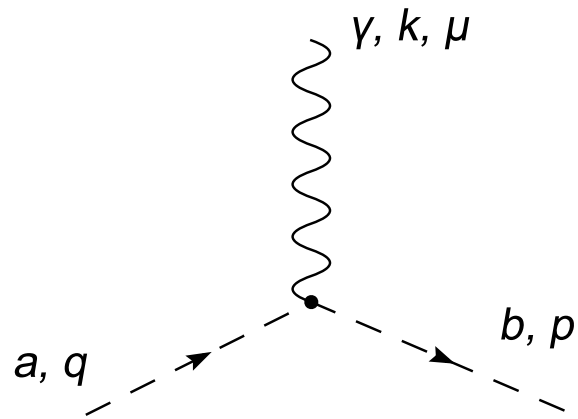


Figure 28: photon-ghosts(gluon)

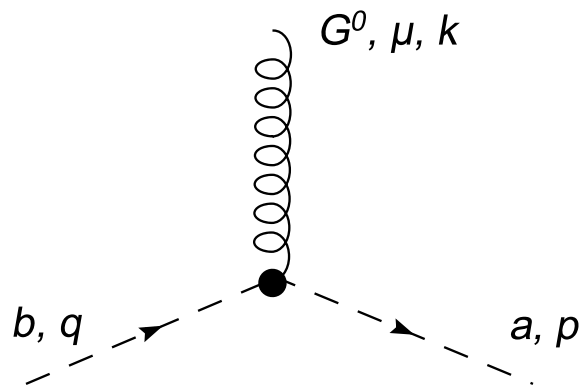


Figure 29: G^0 -ghosts(gluon)

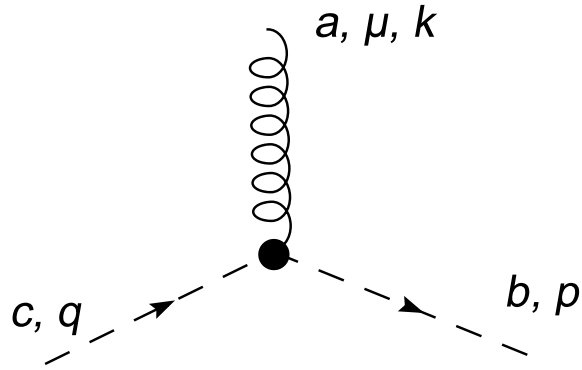


Figure 30: gluon - ghost(gluon)

gluon-ghosts(gluon) vertex

$$-g_3 p_\mu (f_{abc} \cos(p \times q) - d_{abc} \sin(p \times q)) \quad (\text{B.111})$$

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